3D magnetic scalar potential finite element formulation for conducting shells coupled with an external circuit

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Abstract — This paper presents a 3D finite element formulation for a skin depth-independent shell element. It can describe thin conducting shells, which are multiply connected (i.e. which have holes), and which can be coupled with an external circuit. This formulation takes into account the field variation through depth due to skin effect. The method is first described. Computations of two numerical examples modelled with the shell element are then presented.

I. INTRODUCTION

Thin conducting shells have a small thickness "e" compared with their other dimensions. When these shells are meshed with volume elements, a large number of them is needed to avoid poor aspect ratio elements, which lead to numerical errors. In thin conducting shells, skin depth can be much thinner than thickness e, when frequency is high. In that case, mesh problems are increased. Shell element formulations which are independent in terms of skin depth have been developed using the magnetic scalar potential ϕ with the Boundary Integral Method in 3D by Krähenbühl [1], and with the 3D Finite Element Method by Mayergoyz [2] and by us in a previous work [3]. However, these formulations cannot describe multiply connected shells, i.e. with holes. In this paper, we extend the formulation to take into account such multiply connected shells and the coupling with an external circuit.

The method to obtain the shell element formulation is first presented. Then, computations of two numerical examples with simple geometry and modelled with the shell element, are presented.

II. NUMERICAL METHOD

A. Typical problem to solve

Let Ω_c be the thin conducting shell, Γ_1, respectively Γ_2, the boundary between this shell Ω_c and air region Ω_1, respectively Ω_2 (cf. fig. 1). Let $\Omega_a = \Omega_1 \cup \Omega_2$ be the non conducting region (« a » for air). The shell can have one or several holes and can have terminals coupled with an external circuit. There can also be coils. The $\mathbf{t}_0$-ϕ magnetic scalar potential formulation is used in the air and in the conducting shell [4] [5], where $\mathbf{t}_0$ is the source field due either to the current in coils or to the current flowing in the shell between two of its terminals or around holes. Since there is a jump in the tangential component of the magnetic field, a double layer node surface element is needed to describe the shell. The nodes of each couple of nodes are at the same coordinates.

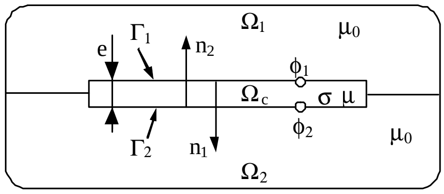

Fig. 1. Typical problem and notations

B. Surface impedance boundary conditions for a shell

The surface impedance boundary conditions for a shell link tangential magnetic fields $\mathbf{h}_{1s}$ and $\mathbf{h}_{2s}$ to electric fields $\mathbf{e}_1$ and $\mathbf{e}_2$ of both sides of this shell. Their expressions can be found in [2] [3]:

$$\begin{aligned} \mathbf{e}_1 &= \mathbf{n}_1 \times \left(Z_{12}\mathbf{h}_{2s} - Z_{11}\mathbf{h}_{1s} \right) \\ \mathbf{e}_2 &= \mathbf{n}_2 \times \left(Z_{21}\mathbf{h}_{1s} - Z_{22}\mathbf{h}_{2s} \right) \end{aligned} \qquad (1)$$

For a shell of thickness e with a linear material of permeability μ and conductivity σ, the surface impedances are (δ: skin depth, $\delta=(2/\omega\mu\sigma)^{0.5}$, ω: pulsation, $\omega=2\pi f$):

$$Z_{11} = Z_{22} = \frac{a}{\sigma \mathrm{th}(ae)} \; ; \; Z_{12} = Z_{21} = \frac{a}{\sigma\, \mathrm{sh}(ae)} \; ; a = \frac{1+j}{\delta} \; .$$

In this linear case, magnetic and electric fields have an exponential variation through thickness of the shell.

C. Voltage-current relation for shells

The voltage current relation for a bulk solid conductor, where u_k corresponds to the voltage drop between two terminals of this conductor or around a hole, is written [4]:

$$u_k = \int_{\Omega_c} \mathbf{j}_{0k} \cdot \mathbf{e}\, d\Omega + \int_{\Omega} \mathbf{t}_{0k} \cdot \frac{d\mathbf{b}}{dt} d\Omega \; . \qquad (2)$$

where $\mathbf{j}_{0k}$ is the current density in the conductor with a current of 1 A applied between the two terminals or flowing around a hole, and $\mathbf{t}_{0k}$ is the source magnetic field due to this $\mathbf{j}_{0k}$ current density. For a hole, u_k is set to 0 in (2). To take into account the conducting shell, this relation (2) is transformed applying the same approach as in [5], using surface impedance boundary conditions (1):

$$\begin{aligned} u_k = j\omega \int_{\Omega_a} \mathbf{t}_{0k} \cdot \mathbf{b}\, d\Omega + \int_{\Gamma} \big[& \mathbf{t}_{01sk} \cdot \left(Z_{11}\, \mathbf{h}_{1s} - Z_{12}\, \mathbf{h}_{2s} \right) \\ & + \mathbf{t}_{02sk} \cdot \left(Z_{22}\, \mathbf{h}_{2s} - Z_{21}\, \mathbf{h}_{1s} \right) \big] d\Gamma \end{aligned} \qquad (3)$$

where $\mathbf{t}_{01sk}$ and $\mathbf{t}_{02sk}$ are the tangential source fields $\mathbf{t}_{0k}$ at both sides of the shell. Their values are different due to the current density $\mathbf{j}_{0k}$ flowing in the shell.

D. Voltage-current relation for coils and shell

The voltage current relation for a stranded coil number k with voltage u_k and current i_k, is written [5]:

$$u_k = R_k\, i_k + \int_{\Omega} \mathbf{t}_{0k} \cdot \frac{d\mathbf{b}}{dt} d\Omega\,. \quad (4)$$

To take into account the conducting shell, this relation is transformed applying again the same approach as in [5], using surface impedance boundary conditions (1):

$$u_k = R_k\, i_k + j\omega \int_{\Omega_a} \mathbf{t}_{0k} \cdot \mathbf{b}\, d\Omega + \int_{\Gamma} [\mathbf{t}_{01sk} \cdot (Z_{11}\,\mathbf{h}_{1s} - Z_{12}\,\mathbf{h}_{2s}) + \mathbf{t}_{02sk} \cdot (Z_{22}\,\mathbf{h}_{2s} - Z_{21}\,\mathbf{h}_{1s})] d\Gamma \quad (5)$$

E. *Finite element formulation for shells*

The magnetic field **h** is expressed as follows:

$$\mathbf{h} = \sum i_{bk} \mathbf{t}_{0k} + \sum i_{ck} \mathbf{t}_{0k} - \mathbf{grad}\phi \qquad \text{in } \Omega_a\,, \quad (6)$$

$$\mathbf{h}_{ms} = \sum i_{bk} \mathbf{t}_{0ms_k} + \sum i_{ck} \mathbf{t}_{0ms_k} - \mathbf{grad}_s \phi_m \quad \text{on } \Gamma\,. \quad (7)$$

where m in (7) corresponds to the side number of the shell ($m \in \{1,2\}$), i_{bk} are the current unknowns corresponding to the coils and i_{ck} are the current unknowns corresponding to the holes of the shell (one i_{ck} unknown per hole) and to the couples of terminals of the shell (n-1 i_{ck} unknowns when the shell has n terminals). When coupling relations (3) and (5) to the $\mathbf{t}_0$-ϕ formulation for shells [2] [3] and using (6) and (7), we get a symmetrical system of equations, with unknowns ϕ, i_{bk} and i_{ck}. The presented formulation has been developed in the Flux® software [6].

III. Numerical Examples

The formulation presented above has been tested on several test-cases. Two of them are presented below.

A. *Rectangular loop shell*

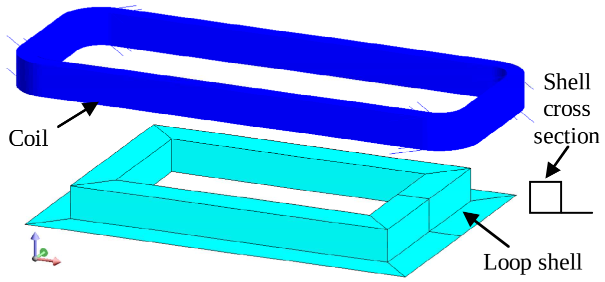

Fig. 2. Rectangular loop shell and coil

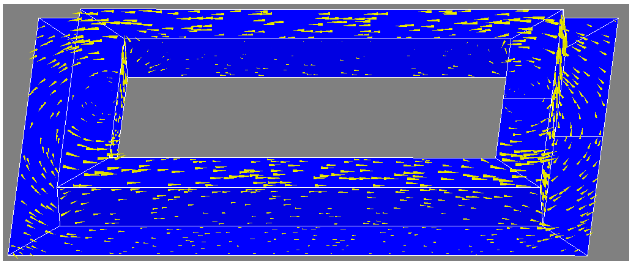

Fig. 3. Arrows of current density (imaginary part)

The first numerical example is a thin conducting rectangular loop shell with a “d”-shape cross section, submitted to the field of a coil supplied by a 50 Hz AC current (cf. figure 2). The loop shell is 0.6 mm thick, 120 cm long, 80 cm large (resistivity = 2×10^{-7} Ωm, $\mu_r = 300$, $\delta = 1.83$ mm). The loop shell has one hole, so there is one additional unknown i_{ck} in the system of equations and the corresponding equation is relation (3). On figure 3 are depicted the arrows of the current density.

B. *Squared sheet iron and wire*

The second numerical example consists of a 100 kHz AC current source which supplies a round copper wire connected to a square sheet iron in which the current returns to the current source. The sheet iron has been described by the shell element formulation presented in part II (thickness e = 2 mm, length: 1 m, resistivity: 1.7×10^{-7} Ωm, $\mu_r = 500$, $\delta = 29.3$ µm). This sheet iron has two circular terminals which correspond to relation (3) in the system of equations. The wire is represented by a coil which is not meshed and corresponds to relation (5).

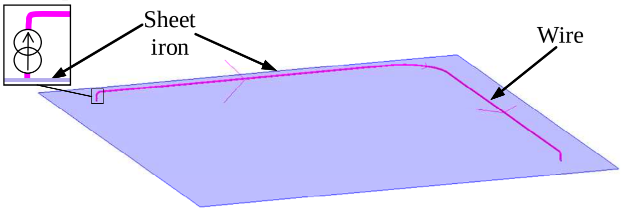

Fig. 4. Squared sheet iron and wire supplied by a current source

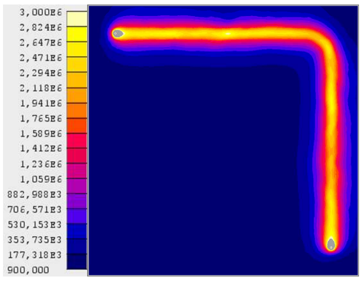

Fig. 5. Isovalues of current density on top surface of sheet iron

The current density is flowing in the sheet iron under the wire, from one terminal to the other (cf. figure 5).

IV. References

[1] L. Krähenbühl and D. Muller, "Thin layers in electrical engineering. Example of shell models in analysing eddy-currents by boundary and finite element methods," *IEEE Trans. Magn.*, Vol. 29, No 2, pp. 1450-1455, March 1993.

[2] I.D. Mayergoyz and G. Bedrosian, “On calculation of 3-D eddy currents in conducting and magnetic shells,” *IEEE Trans. Magn.*, vol.31, no.3, pp. 1319-1324, 1995.

[3] C. Guérin, G. Tanneau, G. Meunier, P. Labie, T. Ngnegueu and M. Sacotte, “A shell element for computing 3D eddy currents-application to transformers,” *IEEE Trans. Magn.*, vol. 31, no.3, pp. 1960-1963, May 1995.

[4] G. Meunier, Y. Le Floch and C. Guérin, « A non linear circuit coupled t-t0-ϕ formulation for solid conductor », IEEE Trans. Magn., vol. 39, No. 3, p. 1729–1732, Mai 2003.

[5] G. Meunier, C. Guérin and Y. Le Floch, “Circuit-coupled t0-ϕ formulation with surface impedance condition”, *IEEE Trans. Magn.*, vol. 44, no.6, pp. 730-733, June 2008.

[6] Flux®, CEDRAT, www.cedrat.com